CONVECTIVE STABILITY IN A SYSTEM OF TWO INFINITE HORIZONTAL LAYERS OF IMMISCIBLE LIQUIDS

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A numerical analysis is offered of convective stability in a system of two adjacent horizontal layers of immiscible liquid.

Underground cavities are used for long-term storage of large quantities of liquid fuel. The volume is initially filled with either natural gas under high pressure or a brine consisting of a concentrated solution of common salt in water.

Heat-transfer processes in the system of two adjacent immiscible liquids created by filling the cavity with fuel are quite complex and require detailed study. In particular, one of the important problems is the study of conditions for development of convective liquid motion in such a system.

The study of hydrodynamic stability of liquid motion with natural convection in such a system must commence with the classical formulation of the problem of layers infinite in the horizontal plane, bounded by rigid horizontal surfaces. The problem is formulated as a conjugate one, in the sense of [1].

1. We will consider a system of two horizontal liquid layers with a division between them (the liquids do not mix), with the layer of lighter liquid located above. The external boundaries are rigid surfaces maintained at constant temperatures, the temperature of the lower lamina T_{01} being higher than that of the upper T_{02} ($\delta T_0 = T_{01} - T_{02} > 0$). We choose a vertical axis directed upward as the z axis, with origin at the lower boundary. Under these conditions, each layer will have its own constant temperature gradient, directed downward. Consequently, at certain temperature differentials δT_0 mechanical liquid equilibrium may prove unstable with respect to some perturbation, the development of which over time may lead to convection.

We will consider the stability of equilibrium with reference to small perturbations (linear-stability-theory approximation). In this approximation small perturbations of velocity $\vec{v}_j\{u_j, v_j, w_j\}$ and temperature T satisfy the following linear equations ([2], Chap. 1):

$$\frac{\partial \vec{v}_j}{\partial t} = -\frac{1}{\rho_{0j}}\nabla p_j + \nu_j \Delta \vec{v}_j + g\beta T_j \vec{r},$$

$$\frac{\partial T_j}{\partial t} + w_j \frac{dT_j^0}{dz} = a_j \Delta T_j, \qquad (1)$$

$$\operatorname{div} \vec{v}_j = 0,$$

where the index j takes on the values 1, 2, and quantities with indices 1, 2 refer to the lower and upper layers, respectively; $\vec{r}$ is a unit vector directed upward along z.

We will write boundary conditions for system (1). At the solid boundaries for z = 0, H

$$u_j = v_j = w_j = 0; \quad T_j = 0 \quad (j = 1, 2). \qquad (2)$$

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At the division between the liquids

$$w_1 = w_2 = 0; \quad u_1 = u_2; \quad v_1 = v_2;$$
$$\mu_1 \frac{\partial u_1}{\partial z} = \mu_2 \frac{\partial u_2}{\partial z}; \quad \mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z}; \tag{3}$$
$$T_1 = T_2; \quad \lambda_1 \frac{\partial T_1}{\partial z} = \lambda_2 \frac{\partial T_2}{\partial z}.$$

The condition for w_j (j = 1, 2) follows from the requirement of impermeability of the liquid interface, and the conditions for the components u_j, v_j (j = 1, 2) from the requirement of equality of tangent velocity components and stresses at the interface.

In studying normal perturbations ($\sim e^{\sigma t}$ and periodic in the horizontal plane), one usually eliminates pressure p and components u, v from Eq. (1), obtaining one fourth-order equation in the component w. For the existence of solutions allowing separation of the variables obtained in a system of equations in T and w, it is necessary that the condition $\Delta_1 f + k^2 f = 0$ be satisfied, where Δ_1 is the Laplacian in the plane (x, y), and f is the temperature, or the vertical velocity component, $k^2 = k_1^2 + k_2^2$. We note that in this case the conditions do not introduce into the boundary problem separate wave numbers k_1^2 and k_2^2, defining the periodicity of the perturbations along the horizontal axes x and y. For this reason the eigennumbers of the problem, and also the critical Rayleigh number, are determined by the parameter k^2, and the ratio between k_1^2 and k_2^2 remains arbitrary. Thus, to one and the same critical Rayleigh number there corresponds an infinite set of perturbations. In particular, there will exist two-dimensional (planar) motions in the form of convective swells with axes parallel, for example, to the y axis ($k_2^2 = 0$, $k_1^2 = k^2$). It is this latter type of perturbation that we will consider further. Consideration of normal perturbations of the two-dimensional convective swell type, while not destroying the generality of the results relative to critical parameters, permits reduction of Eq. (1) to equations for temperature and current function in the plane (x, y), since for the perturbations considered velocity components v will be absent from the motion equation.

If we introduce the change in variables $u = -\partial\psi/\partial z$, $w = \partial\psi/\partial x$ and eliminate pressure in the motion equations (1), then in dimensionless form the equations for perturbations ψ_j, T_j (j = 1, 2) will be written in the form

$$\frac{1}{\mathrm{Pr}_1} \frac{\partial}{\partial \tau} \Delta\psi_1 = \Delta^2\psi_1 + R_1 \frac{\partial T_1}{\partial x},$$
$$\frac{\partial T_1}{\partial \tau} + \frac{dT_1^0}{dz} \frac{\partial \psi_1}{\partial x} = \Delta T_1; \tag{4}$$
$$\frac{1}{\mathrm{Pr}_1} \frac{\partial}{\partial \tau} \Delta\psi_2 = \nu_{21}\Delta^2\psi_2 + \beta_{21} R_1 \frac{\partial T_2}{\partial x},$$
$$\frac{\partial T_2}{\partial \tau} + \frac{dT_2^0}{dz} \frac{\partial \psi_2}{\partial x} = \Delta T_2. \tag{5}$$

Here we introduce the notation

$$R_1 = \frac{g\beta_1 \delta T_0 H^3}{a_1 \nu_1}; \quad \mathrm{Pr}_1 = \frac{\nu_1}{a_1}; \quad \nu_{21} = \frac{\nu_2}{\nu_1}; \quad \beta_{21} = \frac{\beta_2}{\beta_1}; \quad a_{21} = \frac{a_2}{a_1}.$$

As characteristic units H, H^2/a_1, a_1/H, δT_0 are chosen for length, time, velocity, and temperature, respectively; dT_j^0/dz (j = 1, 2) is the gradient of the unperturbed temperature distribution, defined from the equation $d^2T_j^0/dz^2 = 0$ (j = 1, 2), with the following boundary and conjugate conditions at the interface:

$$\frac{dT_1^0}{dz} = -\frac{\lambda_{21}}{1 + l_H(\lambda_{21} - 1)}; \quad \frac{dT_2^0}{dz} = -\frac{1}{1 + l_H(\lambda_{21} - 1)}. \tag{6}$$

We write boundary conditions for Eqs. (4), (5). At the rigid boundaries

$$\frac{\partial \psi_1}{\partial z} = \frac{\partial \psi_1}{\partial x} = 0, \quad \frac{\partial \psi_2}{\partial z} = \frac{\partial \psi_2}{\partial x} = 0,$$
$$T_1 = 0 \text{ at } z = 0; \quad T_2 = 0 \text{ at } z = 1, \tag{7}$$

and at the liquid interface for $z = l_L$,

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} = 0; \quad \frac{\partial \psi_1}{\partial z} = \frac{\partial \psi_2}{\partial z}; \quad \frac{\partial^2 \psi_1}{\partial z^2} = \mu_{21} \frac{\partial^2 \psi_2}{\partial z^2};$$
$$T_1 = T_2; \quad \frac{\partial T_1}{\partial z} = \lambda_{21} \frac{\partial T_2}{\partial z}; \quad \mu_{21} = \mu_2/\mu_1; \quad \lambda_{21} = \lambda_2/\lambda_1. \tag{8}$$

If the solution is taken periodic in x in the form

$$\psi_j = i\Psi_j(z, \tau)\exp(ikx), \quad T_j = \theta_j(z, \tau)\exp(ikx) \ (j = 1, 2), \tag{9}$$

then for the amplitude $\Psi_j(z, \tau)$, $\theta_j(z, \tau)$ from Eqs. (4)-(8) we obtain

$$\frac{1}{\mathrm{Pr}_1} \frac{\partial}{\partial \tau} D\Psi_1 = D^2\Psi_1 + kR_1\theta_1,$$
$$\frac{\partial \theta_1}{\partial \tau} - k \frac{dT_1^0}{dz} \Psi_1 = D\theta_1, \tag{10}$$
$$D = \frac{\partial^2}{\partial z^2} - k^2;$$

$$\frac{1}{\mathrm{Pr}_1} \frac{\partial}{\partial \tau} D\Psi_2 = \nu_{21} D^2\Psi_2 + \beta_{21} kR_j\theta_2,$$
$$\frac{\partial \theta_2}{\partial \tau} - k \frac{dT_2^0}{dz} \Psi_2 = a_{21} D\theta_2 \tag{11}$$

and boundary conditions on the rigid boundaries,

$$\Psi_1 = \frac{\partial \Psi_1}{\partial z} = 0, \quad \theta_1 = 0 \quad \text{at} \quad z = 0;$$
$$\Psi_2 = \frac{\partial \Psi_2}{\partial z} = 0, \quad \theta_2 = 0 \quad \text{at} \quad z = 1; \tag{12}$$

and at the interface for $z = l_L$,

$$\Psi_1 = \Psi_2 = 0; \quad \frac{\partial \Psi_1}{\partial z} = \frac{\partial \Psi_2}{\partial z}; \quad \frac{\partial^2 \Psi_1}{\partial z^2} = \mu_{21} \frac{\partial^2 \Psi_2}{\partial z^2};$$
$$\theta_1 = \theta_2; \quad \frac{\partial \theta_1}{\partial z} = \lambda_{21} \frac{\partial \theta_2}{\partial z}. \tag{13}$$

For purely computational reasons [3] we replace the first fourth-order equation (10), (11) by an equivalent system of two second-order equations, introducing the auxiliary turbulence function

$$\Phi(\tau, z) = \left(\frac{\partial^2}{\partial z^2} - k^2\right) \Psi(\tau, z). \tag{14}$$

Utilizing Eq. (14), it will be convenient to write Eqs. (10), (11) in matrix form:

$$A_j\vec{X}_j - B \frac{\partial \vec{X}_j}{\partial \tau} = 0, \quad \vec{X}_j = \vec{X}(\Psi_j, \Phi_j, \theta_j) \ (j = 1, 2),$$

$$A_1 = \begin{bmatrix} -D & 1 & 0 \\ 0 & D & kR_1 \\ k\frac{dT_1^0}{dz} & 0 & D \end{bmatrix}; \quad A_2 = \begin{bmatrix} -D & 1 & 0 \\ 0 & \nu_{21}D & \beta_{21}kR_1 \\ k\frac{dT_2^0}{dz} & 0 & a_{21}D \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\mathrm{Pr}_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{15}$$

From the boundary conditions (13) it is simple to obtain the conjugate conditions for the function Φ (14) in the form

$$\Phi_1 = \mu_{21}\Phi_2 \quad \text{at} \quad z = l_L. \tag{16}$$

If we take the time dependence in Eq. (15) in the form $\exp(\sigma\tau)$, we obtain the problem of finding the eigenvalues,

$$A_j\vec{X}_j - \sigma B\vec{X}_j = 0 \quad (j = 1, 2) \tag{17}$$

with boundary conditions (12), (13), and (16), while the components of the vector $\vec{X}$ in this equation will be functions only of the coordinate z.

2. The problem of determining the eigenvalues of Eq. (17) with boundary conditions (12), (13), and (16) will be solved numerically, using the establishment method in determining eigenvectors from Eq. (15), according to [3].

We write the finite-difference problem approximating Eq. (15) in the form

$$\vec{X}_{i+1} + A_l\vec{X}_i^j + \vec{X}_{i-1}^j = B_l\vec{X}_i^{j-1} \tag{18}$$

$$(l = 1, 2; \quad i = 1, 2, 3, \ldots, N-1; \quad j = 1, 2, 3, \ldots);$$

$$A_1 = \begin{bmatrix} -(2 + h^2k^2) & -h^2 & 0 \\ 0 & -\left(2 + h^2k^2 + \dfrac{h^2}{\Pr\Delta\tau}\right) & kh^2R \\ kh^2\dfrac{dT_1^0}{dz} & 0 & -\left(2 + h^2k^2 + \dfrac{h^2}{\Delta\tau}\right) \end{bmatrix};$$

$$A_2 = \begin{bmatrix} -(2 + h^2k^2) & -h^2 & 0 \\ 0 & -\left(2 + h^2k^2 + \dfrac{h^2}{\Pr\Delta\tau\nu_{21}}\right) & \dfrac{kh^2\beta_{21}R}{\nu_{21}} \\ \dfrac{kh^2}{a_{21}}\dfrac{dT_2^0}{dz} & 0 & -\left(2 + h^2k^2 + \dfrac{h^2}{\Delta\tau a_{21}}\right) \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -h^2/\Pr\Delta\tau & 0 \\ 0 & 0 & -h^2/\Delta\tau \end{bmatrix}; \quad B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -h^2/\Pr\Delta\tau\nu_{21} & 0 \\ 0 & 0 & -h^2/\Delta\tau a_{21} \end{bmatrix}.$$

Here h is the coordinate increment; $\Delta\tau$ is the time increment; the index i indicates change in coordinate, and the index j indicates change in time.

In obtaining the finite-difference equation (18) with second-order approximation of the differential operators a four-point model, two-layered in time, was employed.

To solve Eq. (18) with corresponding finite-difference boundary conditions we use the matrix function

$$\vec{X}_{i-1}^j = P_{i-1}\vec{X}_i^j + \vec{Q}_{i-1}^j \tag{19}$$

$$(i = 1, 2, 3, \ldots, N-1; \quad j = 1, 2, 3, \ldots).$$

Recurrence relationships for the drive matrices P_i and vectors $\vec{Q}_i^j$ ($i = 1, 2, 3, \ldots, N-1$) are written in the form

$$P_i = -(B_l + P_{i-1})^{-1}; \quad \vec{Q}_i^j = -P_i(B_l\vec{X}_i^{j-1} - \vec{Q}_{i-1}^j)$$

$$(l = 1, 2; \quad i = 1, 2, 3, \ldots, N-1; \quad j = 1, 2, 3, \ldots). \tag{20}$$

To commence computation with the recurrence formulas (20) it is necessary to determine the values P_0 and $\vec{Q}_0^j$ from the boundary conditions (12) at z = 0. Moreover, because of the presence of a coefficient discontinuity surface, to establish continuous drive it is necessary to use Eq. (13) at $z = l_L$ to determine the quantities P_n and $\vec{Q}_n^j$, which will serve as initial values for the upper layer.

For reverse drive with recurrence formula (19) it is necessary to determine $\vec{X}_N^j = \vec{X}(0, \Phi_N^j, 0)$ at the point $i = N-1$ from Eq. (12) for z = 1 and Eq. (19).

It should be noted that in transforming to finite-difference equations for the boundary conditions (12)-(13) the following complications arise. The conditions are such that it is impossible to directly determine Φ from them. The Φ values at boundary points are determined approximately during computation, using Eq. (14) and Ψ values at nodes closest to the boundaries, following [4].

From the finite-difference approximation of the boundary conditions in increment h we obtain the matrix P_0, the vector $\vec{Q}_0^j$, and the vector component Φ_N^j in the following form:

$$P_0 = \begin{bmatrix} 0 & 0 & 0 \\ 3/h^2 & -0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \vec{Q}_0^j = \vec{Q}_0(0,\ 0,\ 0);$$

$$\Phi_N^j = \left(\frac{3}{h^2} q_{N-1}^1 - 0.5 q_{N-1}^2\right) \Big/ \left(1 + 0.5 p_{N-1}^{22} - \frac{3}{h^2} p_{N-1}^{12}\right).$$

From the conjugate condition (13) and the definition of the turbulence function (14) in their second-order approximation in h we obtain the relationship of the matrix P_n and vector $\vec{Q}_n^j$ at the liquid interface in the form

$$P_n = \frac{1}{D} \begin{bmatrix} 0 & 0 & 0 \\ \frac{b_1}{h^2} & -\frac{b_1}{6} & -2\lambda_{21} b \\ -\frac{a_1}{h^2} & \frac{a_1}{6} & 2\lambda_{21} a \end{bmatrix}; \quad \vec{Q}_n^j = \vec{Q}(0,\ q_n^{21},\ q_n^3);$$

$$D = ab_1 - a_1 b; \quad a = \left(\frac{1}{3} - \frac{p_{n-1}^{12}}{h^2} + \frac{p_{n-1}^{22}}{6}\right)\mu_{21} + \frac{1}{3};$$

$$b = -\frac{p_{n-1}^{13}}{h^2} + \frac{p_{n-1}^{23}}{6}; \quad a_1 = -2\mu_{21} p_{n-1}^{32}; \quad b_1 = A_1 - \lambda_{21} A_2 - 2p_{n-1}^{33};$$

$$A_1 = 2 + k^2 h^2 + h^2/\Delta\tau; \quad A_2 = 2 + k^2 h^2 + h^2/a_{21}\Delta\tau;$$

$$q_n^2 = \frac{1}{D}\left\{\frac{b_1}{h^2} q_{n-1}^1 - \frac{b_1}{6} q_{n-1}^2 - 2bq_{n-1}^3 - \frac{bh^2}{\Delta\tau}\left(1 + \frac{\lambda_{21}}{a_{21}}\right)\theta_n^{j-1}\right\};$$

$$q_n^3 = \frac{1}{D}\left\{-\frac{a_1}{h^2} q_{n-1}^1 + \frac{a_1}{6} q_{n-1}^2 + 2aq_{n-1}^3 + \frac{ah^2}{\Delta\tau}\left(1 + \frac{\lambda_{21}}{a_{21}}\right)\theta_n^{j-1}\right\}.$$

Eigenvectors are found on the basis of a completely stabilized multistep exponential method. The process of determining successive approximations in time, using the method employed for solution of Eq. (19), will be denoted by the operator A, so that

$$\vec{X}^j = A\vec{X}^{j-1} \quad (j = 1,\ 2,\ 3,\ \ldots). \tag{21}$$

To find the first three eigenvalues of the finite-difference problem corresponding to Eq. (17) an invariant subspace is constructed, extended over the system of three corresponding eigenvectors (see [3], p. 3). Briefly, the essence of this construction is as follows. Selecting for initial values three arbitrary (it is desirable that they be linearly independent) vectors $\vec{X}_i^0$ (i = 1, 2, 3), each with 3(N + 1) dimensions, we construct, according to Eq. (21), a system of three mutually orthogonal vectors $\vec{X}_i^j$ (i = 1, 2, 3; j = 1, 2, 3, . . .). Orthogonalization and normalization of the vectors in the computation process is performed with the equation

$$N_0 = -\sum_{i=0}^{N}\left[\frac{1}{\mathrm{Pr}}\ \Psi_n(z_i)\Phi_m(z_i) - \theta_n(z_i)\cdot\theta_m(z_i)\right] \tag{22}$$

$$(N = 1/h;\quad n,\ m = 1,\ 2,\ 3).$$

The sequence of vectors thus constructed at $j \to \infty$ has limits $\vec{X}$ (i = 1, 2, 3), which lie in the invariant subspace noted above.

If these limiting vectors are used as a base for the subspace, the induction operator matrix $L\{\alpha_{m,n}\}$, whose eigenvalues define the sought for eigennumbers of the problem, may be constructed as

$$\alpha_{mn} = A(\vec{X}_n, \vec{X}_m) \quad (n, m = 1, 2, 3). \tag{23}$$

It is natural that in numerical computations, according to Eq. (23), as $\vec{X}_n$ (n = 1, 2, 3) the approximations $\vec{X}_n^j$ are taken, with the condition required for stabilization of the iteration process (21).

The eigenvalues of the matrix L coincide with the eigenvalues of the parameter q_i (i = 1, 2, 3) of Eq. (18), which are related to the desired eigenvalues σ_i (i = 1, 2, 3) of the finite-difference problem corresponding to Eq. (17) by the relationship

$$\sigma_i = \frac{q_i - 1}{q_i \Delta\tau} \quad (i = 1, 2, 3). \tag{24}$$

The accuracy of eigenvector determination is controlled by the unbinding rate δ_i (i = 1, 2, 3) of the following and preceding time approximations of these vectors:

$$\delta_i = \sqrt{(\vec{\Delta}_i, \vec{\Delta}_i)}; \quad \vec{\Delta}_i = A\vec{U}_i^j - q_i\vec{U}_i^j \qquad (i = 1, 2, 3; \quad j = 1, 2, 3, \ldots), \tag{25}$$

where $\vec{U}_i^j$ are the eigenvectors of the problem which are linearly independent combinations of the approximations $\vec{X}_i^j$, with the constants in these combinations being components of the corresponding vectors of matrix L.

Numerical solution using the above algorithm was performed on a Minsk-22 M electronic computer.

3. The numerical-analysis method based on the solution algorithm presented above consists of the following. For a given ratio of parameters and fixed ratio of liquid layer thicknesses, the neutral curve R = f(k) is calculated. For a given k and a number of R values we define, according to Eq. (21), three eigenvectors and corresponding eigennumbers (24). Calculations show that the first eigennumber σ_1 is linearly dependent on R over a sufficiently wide range. Thus, the neutral R value for a given k is found by linear interpolation to $\sigma_1 = 0$. By quadratic interpolation of the neutral-curve results the critical values R* and k* are determined (Table 1).

The problem formulated, Eqs. (10)-(14), contains a set of particular cases because there enter into the equations, aside from R and k, six more arbitrary parameters – the ratios of the physical properties of the liquids in the system.

This present study will offer results of analysis of a brine (common salt solution) –kerosene system with the following parameters: Pr = 10; $\lambda_{21} = 0.2$; $\mu_{21} = 0.8$; $\nu_{21} = 1.17$; $a_{21} = 0.48$; $\beta_{21} = 3$.

Figure 1 shows the critical Rayleigh criterion R* as a function of relative depth of the lower liquid layer l_L. It is evident that with increase in l_L the R* values increase without limit, i.e., with decrease in relative kerosene depth the hydrodynamic stability of the system (at least with respect to infinitely small perturbations) increases. With decrease in l_L the criterion R* asymptotically approaches a certain finite value (R* ~ 74). The wave number k behaves in approximately the same manner relative to liquid depth l_L.

The results obtained are of undoubted practical interest. The physical basis of these results may be explained as follows. From Eq. (6) it follows that the ratio of the undisturbed temperature-distribution gradients in the brine layer and kerosene is equal to $(dT_1^0/dz)/(dT_2^0/dz) = \lambda_{21}$. With the numerical parameter values in the case considered this ratio equals 0.2, i.e., in the kerosene layer the temperature gradient is five times higher than in the brine layer. Therefore, for a given temperature differential between the external boundaries, conditions for development of a volume force counteracting the force of viscous resistance are more favorable in the kerosene layer. Consequently, it can be expected that perturbations determining the development of instability in the entire system will be those arising in the kerosene layer and characterized by its parameters.

If we define a Rayleigh criterion for the kerosene layer as $R_2 = g\beta_2(dT_2^0/dz)(H-h)^4/a_2\nu_2$, then R_1 may be written in the form

$$R_1 = \frac{a_{21}\nu_{21}}{\beta_{21}} \cdot \frac{1 - l_L(1-\lambda_{21})}{(1-l_L)^4} R_2, \tag{26}$$

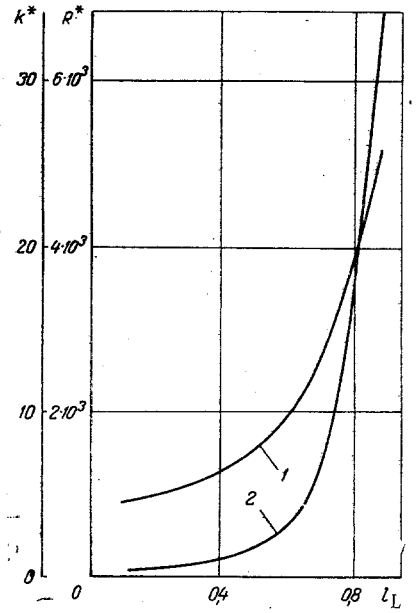

Fig. 1. k* (1) and R* (2) vs l_L.

or, substituting numerical data for the parameters in our case, we obtain

$$R_1 = 0{,}188\,\frac{1-0{,}8l_L}{(1-l_L)^4}\,R_2.$$

From this it follows that R_1^* varies with l_L in a fourth-order parabolic manner, under the condition, naturally, that R_2^* depends weakly on l_L, which, as follows from the observations above, occurs in our case. Figure 1 agrees with this interpretation.

In an analogous manner it may be found that the wave number k_1 is related to the wave number k_2 characterizing the periodicity of perturbations in the kerosene layer by the formula

$$k_1 = \frac{k_2}{1-l_L}. \qquad (27)$$

The dependence of k_1 on l_L in Fig. 1 agrees with Eq. (27).

For the nonlinear instability problem in the case of convective heat transfer in a horizontal layer of a homogeneous liquid, a method was developed in [5] giving the limiting stable amplitude of convective currents for a certain Rayleigh number interval, above the critical value. It was shown that an infinitely small perturbation, corresponding in the linear theory to the first eigennumber $\sigma_1 > 0$ for the critical wave number k* and criterion R > R* after initial exponential growth will tend to an equilibrium perturbation of finite amplitude.

It then follows that in our case finite perturbations which might be established in the system considered will coincide in form with the first eigenvector of the corresponding linear theory problem. In

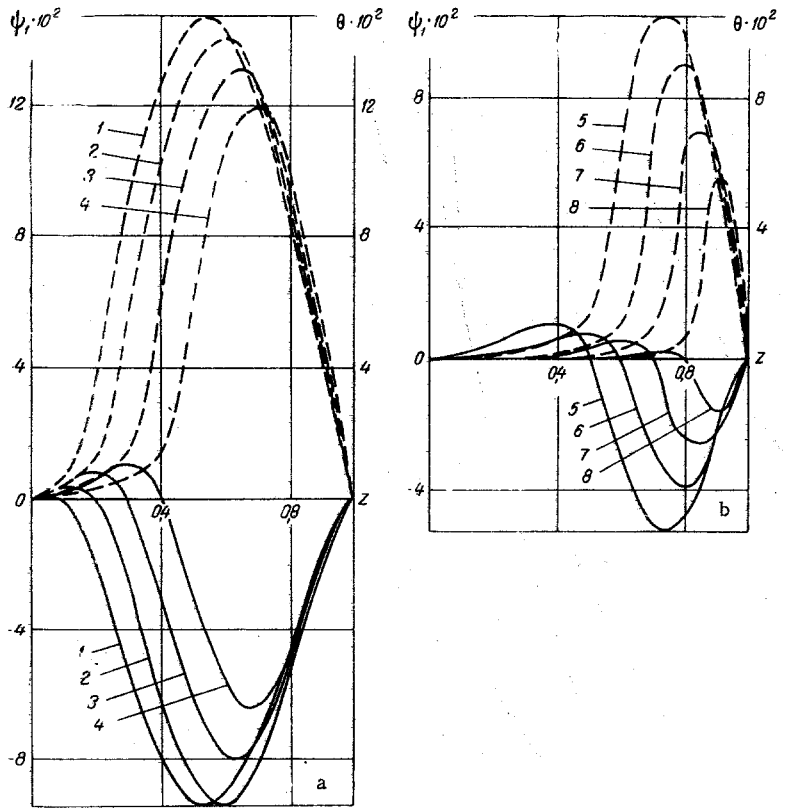

Fig. 2. Current function amplitude (solid lines) and temperature (dashed) for various l_L: 1) l_L = 0.1; 2) 0.2; 3) 0.3; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.7; 8) 0.8.

TABLE 1. Neutral Curves for Various Interface Positions l_L

k	R	k	R	k	R	k	R
$l_L=0,1$		$l_L=0,4$		9,9	605,0	$l_L=0,85$	
3	138,9	6,2	236,8	10	605,5	25	7062,4
4	107,3	6,5	235,7	10,5	610,1	25,4	7053,4
5	104,9	6,6	236,2	11	619,6	25,5	7052,1
5,5	109,6	6,7	236,2	$l_L=0,7$		25,6	7051,1
6	116,8	$l_L=0,5$		11	1262	25,7	7050,4
6,5	126,4	7,5	358,4	11,5	1238,2	25,8	7050,0
$l_L=0,2$		7,6	358,0	12	1223,3	26	7050,3
4	142,5	7,7	357,7	12,5	1214,7	27	7070,3
5	129,4	7,8	357,7	13	1212,5	$l_L=0,9$	
5,1	129,3	7,9	357,8	13,5	1215,9	38	20808
5,2	129,4	8,0	358,1	$l_L=0,8$		38,4	20799
6	135,4	8,1	358,5	17	3437	38,6	20797
$l_L=0,3$		8,2	359,2	18	3375,6	38,8	20796
3	312,6	8,3	360,0	18,5	3358,8	39,0	20797
4	206,6	$l_L=0,6$		19	3348,9	39,2	20800
5	173,7	8,5	620,6	19,5	3347,3	39,4	20804
5,6	169,3	9	610,0	20	3352,0	40,0	20828
5,7	169,2	9,5	605,1	20,5	3363,0	41,0	20897
5,8	169,3	9,6	604,7	21	3379,9		
5,9	169,5	9,7	604,6				
7	181,0	9,8	604,7				

Fig. 2a, b curves of the amplitude of the current function and temperature as functions of the vertical coordinate are shown for various interface positions. It is evident that the main perturbation turbulence (relative to high intensity) is localized in the kerosene layer, while the turbulence of opposite rotation and significantly lower intensity in the brine layer localized in the region near the kerosene as layer thickness increases. This also supports the interpretation of Fig. 1 given above.

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